STOCHASTIC DYNAMICS ON CONSTRAINED SPACES AND THEIR APPLICATIONS

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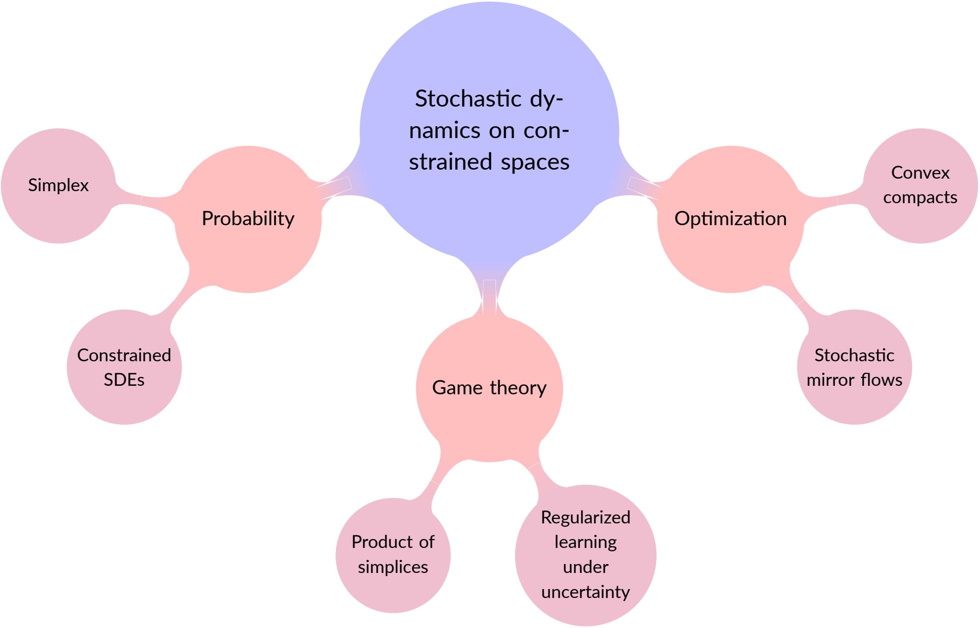
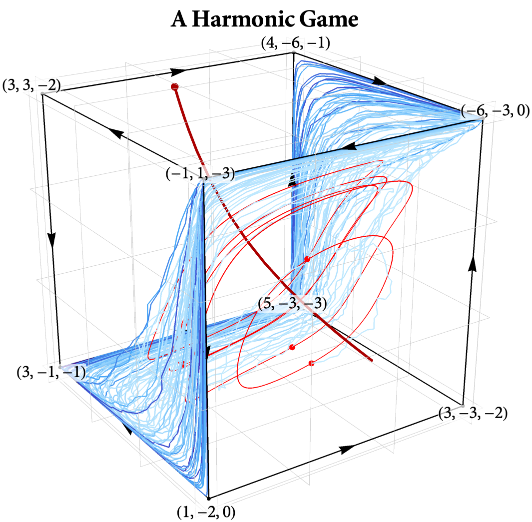
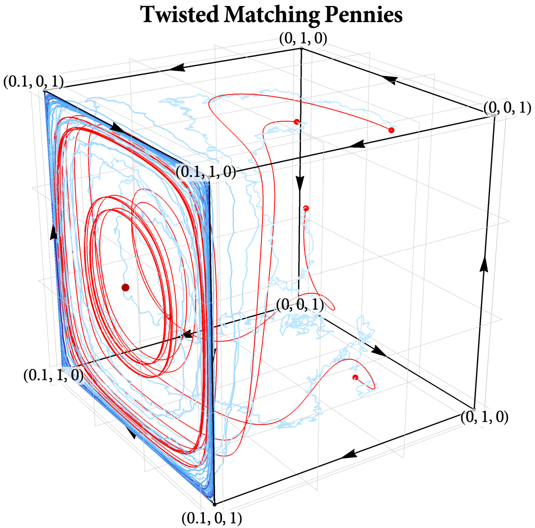
$\langle$ GHOST Day | September 30, 2025 $\rangle$

General overview

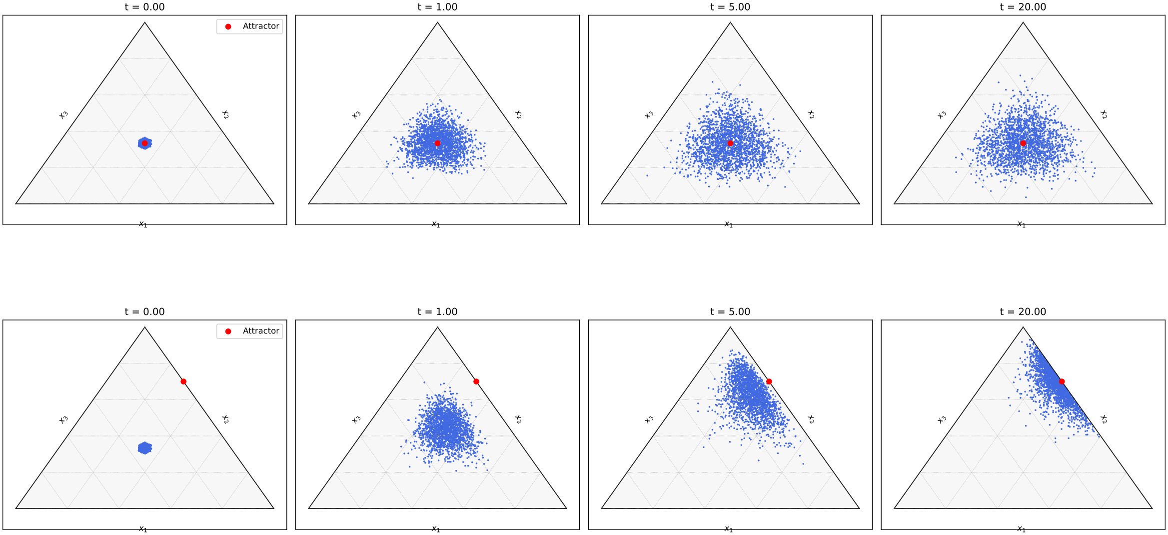


Examples and figures

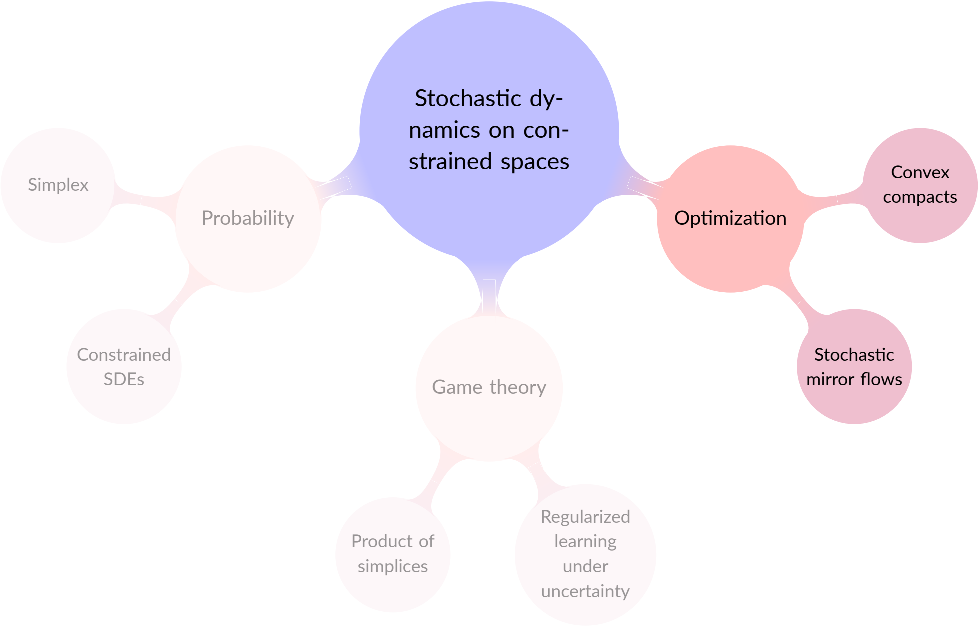




Examples and figures, cont'd



General overview



Setup and mirror flow

Goal: minimize $f(x)$ subject to $x \in \mathcal{X}$

- ☞ $\mathcal{X} \subset \mathbb{R}^d$ convex and compact
- ☞ $f : \mathcal{X} \to \mathbb{R}$ strongly convex and L-smooth

Mirror flow:

$$\dot{y}_t = -\nabla f(x_t)$$
$$x_t = Q(\eta_t y_t) \qquad \# \, Q : \mathcal{Y} \to \mathcal{X} \text{ "projection-like" mapping}$$

Convergence result: trajectories x_t converge to the unique minimum x^* of f

What happens when ∇f is only known up to some random error?

Previous works

Stochastic mirror flow:

$$\begin{aligned} dY_t &= -\nabla f(X_t)dt + \sigma dW_t \\ X_t &= Q(\eta_t Y_t) \end{aligned} \qquad \text{(S-MF)}$$

☞ **Stochastic** differential equation with **Brownian noise**

$\sim$ perturbed by Gaussian noise at each (infinitesimal) time step

Main result: $\bar{X}_t := \frac{1}{t}\int_0^t X_s ds \to x^*$ (a.s.) when $\eta_t \searrow 0$.

Drawbacks:

- Noise process is **continuous in time** $\rightsquigarrow$ cannot model **abrupt shocks**
- Noise process admits **exponential moments** $\rightsquigarrow$ cannot model **heavy-tailed** noise

Our results

Lévy mirror flow:

$$\begin{aligned} dY_t &= -\nabla f(X_t)dt + dL_t \\ X_t &= Q(\eta_t Y_t) \end{aligned} \tag{L-MF}$$

- ☞ SDE with **Lévy noise** $\rightsquigarrow$ can be **discontinuous** and **heavy-tailed**
- ☞ **Ex:** Brownian motion, Poisson processes, stable processes, ...

Noise assumption: L_t is **centered** and admits moments up to order $p \in (1, 2]$

Main result: $\bar{X}_t := \frac{1}{t}\int_0^t X_s ds \to x^*$ (a.s.) when $\eta_t \searrow 0$

Finite variance is not needed for convergence results!